National
Qualifications
2022 MODIFIED

X803/77/11

Statistics
Paper 1

MONDAY, 23 MAY
9:00 AM – 10:00 AM

Total marks — 30

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

You may refer to the Statistics Advanced Higher Statistical Formulae and Tables.

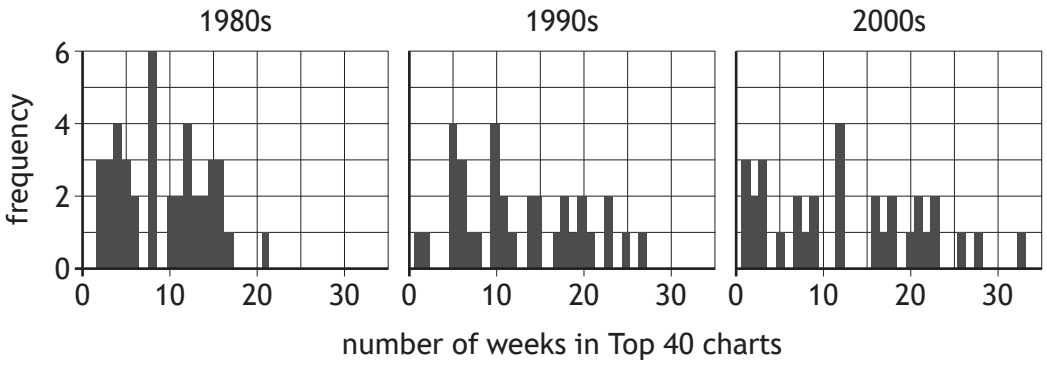
Total marks — 30 **MARKS**

Attempt ALL questions

1. A study looked at the number of weeks that individual songs were in the American Top 40 music charts over recent decades. Due to cost constraints, only a small amount of the data could be accessed. Two per cent of the songs that were in the Top 40 from three decades were obtained by stratified random sampling with each year used as a strata. Computer output of the summary statistics and frequency charts is shown.

Output 1

Decade	Min	Q1	Med	Q3	Max	Mean	SD	n
1980s	2	5	8	13	21	9.293	5.036	41
1990s	1	6	11	18	27	12.700	7.038	33
2000s	1	4	12	20.5	33	12.844	8.991	32



(a) Determine if there were any outliers for the 1980s data set. **2**

(b) Explain how the stratified sample of the songs from the 1980s decade would have been conducted. **2**

(c) By comparing measures of location, spread and sample sizes, across the three decades, describe the emerging trends for the number of weeks a song could expect to be in the Top 40 charts. For each trend, clearly explain what it means in terms of the context of the study. **6**

Confidence intervals for means were calculated using *t*-distributions.

Output 2

Decade	95% CI	df
1980s	(7.70, 10.88)	40
1990s	(10.20, 15.20)	32
2000s	(9.73, 15.96)	31

(d) (i) Write down the calculation that gives the 1990s confidence interval in **Output 2** and state the assumption required for this calculation to be valid. **3**

(ii) Explain what is meant by this confidence interval for someone with no formal knowledge of statistics. **1**

1. (continued)

For the years 2010–2019, a further stratified random sample of two per cent of the songs was obtained, and gave the following summary output.

Output 3

Decade	Min	Q1	Med	Q3	Max	Mean	SD	n
2010s	1	3	15.5	23	42	15.156	12.713	32

A two sample t-test for a difference in population means was then performed using the data on the songs from the 2000s (sample 1) and the 2010s (sample 2). Under the assumption that the lengths of times in the charts are normally distributed, the output of this test is given below.

Output 4

```
sample 1 mean = 12.844, sample 1 SD = 8.991
sample 2 mean = 15.156, sample 2 SD = 12.713
alternative hypothesis: true difference in means is
not equal to 0
pooled SD = 11.010
t = -0.840, df = 62, p-value = 0.4042
```

(e) Interpret the p-value of this output, in the context of this data. **2**

(f) A further assumption associated with this t-test for a difference in population means is that the unknown parent population standard deviations are equal.

Comment on the validity of this assumption in the given context. **1**

[Turn over

2. An extract from a draft report by a researcher is given below.

It is known to contain some flaws and questionable methodology.

Read it and then answer the questions that follow.

Introduction

With the increased use of smartphones, I wondered what data could be sourced and analysed about the number of young people who nowadays wear a wristwatch. I chose to focus my research sample on pupils at my large secondary school.

Method

In my school I chose to ask both pupils and teachers whether they wore a wristwatch and if they did, on which wrist they wore it. Here, a wristwatch was defined as a 'gadget that is worn on the wrist that tells the time of day: so analogue watches, digital watches, smartwatches, activity trackers all count. A smartphone is not a wristwatch.'

I created an online survey that was sent to teachers for years S1 to S5 to ask those pupils who were present one randomly chosen morning in their registration class about wristwatches. They recorded how many pupils were wearing a wristwatch on that day and upon which wrist it was worn. In addition, the teachers recorded the same information for themselves.

The S6 pupils in my school do not have a registration class. Therefore, they were each sent an individual survey by email and asked to respond as soon as possible.

Results

A summary table of the survey results are shown below.

Table 1

Group	Left wrist	Right wrist	No watch	Total responses	Group size	Response rate
S1	62	17	145	224	240	93%
S2	38	28	149	215	232	93%
S3	49	15	167	231	252	92%
S4	46	9	130	185	245	76%
S5	42	12	152	206	234	88%
S6	31	8	62	101	218	46%
Teacher	38	6	16	60	60	100%

Responses were only received from 46% of the S6 pupils, which contrasted with the 88.2% coverage from all S1 to S5 pupils that had come via their registration teachers.

Analysis

There seemed to be two methods that I could analyse this data with: comparing numbers of wristwatch wearers (**Figure 1**) or comparing percentages (**Figure 2**). One might illuminate more than the other.

2. (continued)

Figure 1 Number of wristwatch wearers

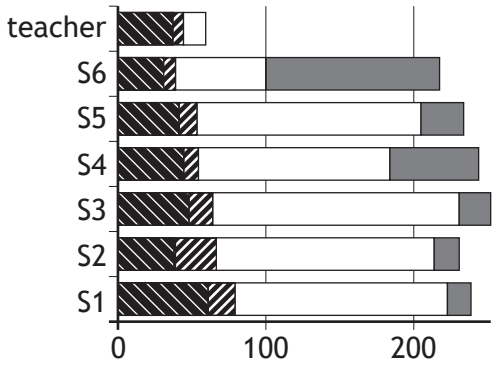
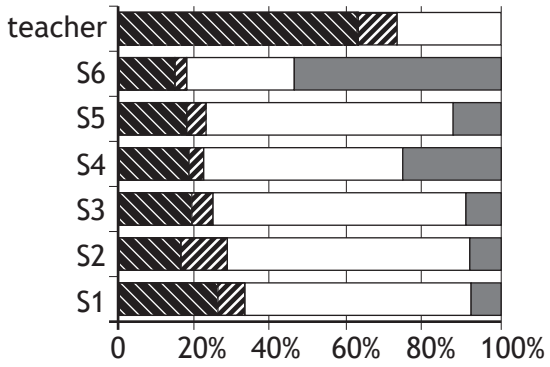
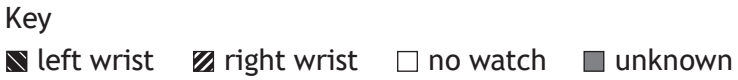


Figure 2 Percentage of wristwatch wearers





Analysis A — Comparing Numbers

I chose to conduct a chi-squared goodness-of-fit test of the numbers of wristwatch wearers against a uniform distribution.

Table 2

Group	S1	S2	S3	S4	S5	S6	Teacher
Observed wearers	79	66	64	55	54	39	44
Expected wearers	57.3	57.3	57.3	57.3	57.3	57.3	57.3

This gave a chi-squared test statistic of 19.5 with 6 degrees of freedom, which is statistically significant at the 5% level. So we could conclude that there is strong evidence that the number of wristwatch wearers was not equal across all year groups.

Analysis B — Comparing Proportions

From **Figure 2**, the poor response rate from S6 and the large amount of data missing for S4 caused me concern were I to compare them. Therefore, looking at the similar sizes of year groups and comparable amounts of missing data, I decided to first compare whether the proportion of S1 pupils who wore a wristwatch was the same as that for S2 pupils.

A two-sample proportion test on 79 out of 224 S1s and 66 out of 215 S2s gave rise to a z-test statistic of 1.02 and a p-value of 0.309, so there was little evidence that they had different proportions at the 5% level of significance.

However, comparing S1 pupils with S5 pupils gave a z-test statistic of 2.03 and a p-value of 0.042, so there was evidence that they had different proportions.

Conclusion

There do appear to be some differences in wristwatch ownership that vary with a young person's age, but these may only be statistically detectable when comparing people of different ages.

MARKS

2. (continued)

(a) Read lines 10 to 14.

State the type of sampling method used and describe a possible disadvantage of this method. **2**

(b) Read lines 19 to 20.

With reference to the methodology used to gather the data, suggest an improvement that should lead to a higher response rate from the S6 pupils. **1**

(c) Compare **Figure 1** to **Figure 2**.

State which group's data appears to be most affected by the change of focus from numbers to percentages and give a reason for this. **2**

(d) (i) Referring to **Table 2**, write down the calculation that generated the number of expected wearers. **1**

(ii) With reference to information in the Introduction, explain why the chi-squared goodness-of-fit test conducted on **Table 2** was not appropriate to involve all 7 groups of people. **1**

(e) Read lines 39 to 40.

State the hypotheses for the test between S1 and S5 pupils, and show the calculations that generate the z-test statistic of 2.03 and the p-value of 0.042. **5**

(f) Read lines 2 to 4, and lines 42 to 44.

The phrasing of the conclusion suggests that the results might hold in the wider population of young people.

Without reference to the sampling method, suggest why you would not be able to extrapolate the conclusions found from the sample to the wider population of young people. **1**

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